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BOOK REVIEWS

SOME RECENT FRENCH VIEWS ON CONCRETE METHODS OF TEACHING MATHEMATICS

In view of the discussions which have been taking place in the meetings of various bodies of teachers of mathematics, a brief account of some recent French utterances on phases of the same general topic will have special interest. The works which I shall cite are: Bertrand, *Les études dans la démocratie* (Paris, 1900, pp. 288); Duclaux, "L'enseignement des mathématiques," *Revue scientifique*, Vol. I (1899), pp. 353-58; Laisant, *L'éducation fondée sur la science* (Paris 1904; pp. xlv+155); Le Bon, *Psychologie de l'éducation* (Paris, 1904; pp. 304); Tannery, J., "L'enseignement de la géométrie élémentaire," *Revue pédagogique*, July, 1903, pp. 1-27. I shall refer to these works simply by the author's name.

The writers all concur in urging with emphasis the teaching of mathematics from concrete beginnings. They are combating what Le Bon calls (p. 251) "the ineradicable habit of the Latin race always to begin with the abstract without first passing through the concrete"—a habit the effects of which can be seen throughout French work in mathematics. It has helped produce that clearness of logical exposition which is so characteristic of French thinking and writing on mathematics, but no one can tell at what cost to the minds of the overwhelming majority of the pupils to whom the doors of mathematics open effectively only through the concrete. This has so far called forth less protest than might have been expected, in part at least because mathematics has designedly been used as the instrument in weeding out the candidates for admission to the highest institutions (Le Bon, p. 248; Tannery, p. 16). But voices are not wanting which urge a wider view of the purpose of the teaching of mathematics and a corresponding modification of the methods of instruction. The discussion will be especially helpful to Americans at the present time, when the consciousness is growing that the "habit of commencing with the abstract" has been far too prevalent here, and when we are earnestly seeking to find and apply the remedy.

The work of Laisant consists of four addresses, written in his customary genial style, and delivered at various times during the last four years. They have also been published separately in the *Revue générale des sciences*. For the purposes of the present paper only the first two addresses, "The Mathematical Initiation" and "The Initiation into the Study of the Physical Sciences," are of interest; the other two, on "Scientific and Psychologic Education" and on "The Problem of Education," not bearing directly on the subject of this paper.

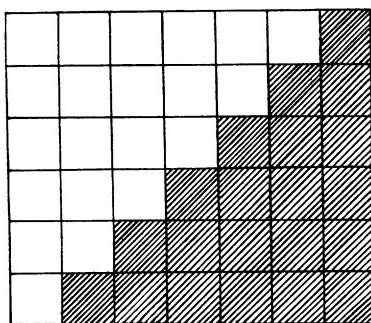
Mr. Laisant sets out with the assertion that even mathematics is an experimental science, that consequently the beginnings of mathematics should be concrete, to be followed later by abstractions, and deplores the fact that the reverse order is at present followed.

I hold that *all the sciences* without exception are experimental, at least to a certain extent; in spite of certain theories which attempt to treat mathematics as a sequence of operations of pure logic based upon pure ideas, it can be asserted that in mathematics, as in all other scientific domains, there exists no notion,

no idea, which could penetrate into our brain without the previous contemplation of the exterior world and of the facts which this world presents to our observation. . . . From this exterior world the first mathematical notions must be obtained, to be succeeded later by abstractions. . . . Now, how are things done today? Primary instruction, so far as it relates to the first notions of arithmetic, seems modeled on that of grammar; and one might just as well say that the latter is modeled on the teaching of arithmetic. That is to say, in the one as in the other the child is taught a number of abstract and confused definitions which he cannot comprehend; under pretext of giving him good, practical directions, he is burdened with a set of rules; and these rules he learns and retains by dint of memory only and applies them thereafter, well or ill, as may be. (Pp. 2, 3.)

After some discussions, relating more particularly to French conditions, comes the following pregnant passage:

The elementary acquisitions in mathematics—and they are of considerable extent—are no less useful, no less indispensable, than the knowledge of reading and writing. I will even add—and this may perhaps seem paradoxical—that these first elements can be assimilated with much less fatigue than the first notions of reading and writing; on one condition always: that is, instead of persevering in the present system of primary teaching, instead of giving an instruction bristling with rules and formulae, appealing to the memory, causing fatigue and producing only disgust, the teaching should be inspired by the philosophic fact that it is necessary in the first place to produce images in the child's brain by means of objects presented to his senses. The instruction should be absolutely concrete, occupied only with the contemplation of external objects, and with the description of these objects; it should continually appear, especially in the primary period, under the form of play, and not of study. (Pp. 6, 7.)



Our author practices what he preaches, and proceeds to give quite a number of concrete illustrations of his meaning, including various uses of squared paper and geometric proofs by cutting, among the latter a simple proof of the Pythagorean theorem. As example, the use of squared paper to find the sum of the first n integers may be cited. The figure alone will suffice. The drawing should be made by the pupil. That the number of unshaded squares is the same as that of the shaded squares is seen by turning the paper bottom side up.

Mr. Laisant justly holds squared paper in high esteem:

Squared paper is a marvelous instrument which ought to be in the hands of whoever works in mathematics, from the family or kindergarten to the polytechnic school and even beyond, and, in a general way, of whoever works in science. But from the pedagogic point of view it is an especially marvelous instrument for giving to young children those first notions of form, of magnitude, and of position, without which the work is a sham. (P. 23.)

He also favors the use of paradoxes to interest the child, such as $\frac{1}{2}$ of $12 = 7$ (by Roman notation) and of "mathematical recreations," an abundance of which can be found in the works of Lucas (French), Ball (English), and others. In the second address he cites (pp. 45 ff.) several works of a similar character for physics and chemistry with special view to what can be done without apparatus. Thus: Tissandier, *La physique sans appareils; la chimie sans laboratoire* (6th ed., 1893); "Tom Tit," *La science amusante*, Vol. III. Perhaps some reader of the *School Review* can give a list of analogous works available in English (on physics, chemistry, mathematics).

Our author does not hesitate to assert that mathematics has a physical, experimental side:

Properly speaking, all sciences are physical, all sciences are experimental. Nevertheless, it has been necessary, in view of the infinite variety of facts which nature presents to us, to make a classification. This was indispensable for threading the labyrinth, but it is neither absolute nor perfect nor eternal, and is far from having the importance generally attributed to it. All the sciences interact and interpenetrate; none has sharply defined boundaries. . . . We live in an immense laboratory—each one of us is a laboratory—the commonest of natural facts which we can contemplate or by which we can be affected represents, so to speak, an infinitude of physical or chemical phenomena superposed. Precisely this superposition blinds us. To know these phenomena precisely, we must be able to isolate them at least relatively, to devise a mode of experimentation which shall make the principal phenomenon predominate so that all the others pass unperceived. This is in the experimental domain an operation a little analogous to mathematical abstraction, and it should be governed by the same principles. (Pp. 37-39.)

The greater difficulty and delicacy of the abstractions of the physical sciences is next pointed out. As to the abstract ideas, definitions, rules, principles of mathematics, Mr. Laisant rightly believes that they will take care of themselves; that the child will make "instinctive abstractions" as needed; that the young child should never be urged to make abstractions which he does not make of himself, and that his attention should not even be drawn to those which he has made (p. 27).

Duclaux takes up particularly the question of geometry. After pointing out the rigidity of the Euclidean method, and the way in which simple things are made difficult by its formalism, he goes on to say (p. 356):

How can this fault be corrected? Simply come back to reality. Put a ruler into the hands of the pupil as soon as the straight line has been defined, compasses as soon as the circle has been defined, a square as soon as he knows what a right angle is, a protractor as soon as he knows what an angle is. When the subtleties demanded by the learned march of the Euclidean method are cast aside, that which we call the first book of Euclid can pass in an hour before the eyes of the pupil who, happy to exercise intelligence, initiative, and even intuition, draws from the joint exercise of these powers confidence in himself and an elation which will stand him in good stead when he encounters real obstacles.

The book of Le Bon takes up some interesting phases of the agitation in France which culminated in the new curricula of 1902. Only the chapter on the teaching of mathematics is of direct interest here (pp. 248-58), and its tenor is sufficiently exemplified by the following quotation:

Mathematics is a language, and acquaintance with it no more develops the intelligence than that of other languages. One does not learn a language to exercise the intelligence, but because it is useful to know. Now, the habit of writing the simplest things in mathematical language is today so widespread that it is necessary for the pupils to learn this language, just as it would be necessary for them to learn Japanese, or Sanskrit, if all the books of science were written in these languages.

The only important thing is to know how one can learn rapidly to comprehend and then to speak this special language of mathematicians. Like those of all languages, the beginnings only of this study, are difficult. They must be made in the most tender infancy, at the same time as reading and writing, but by a method diametrically opposed to that which is used today. The teaching must be by experiences, substituting direct observation of quantities that can be seen and touched for reasoning about symbols. What makes the mathematical instruction of the child so difficult is the ineradicable habit of the Latin race always to begin with the abstract without first passing through the concrete. If ignorance of the psychology of the child were not so widespread and so profound, all the pedagogues would know that the child cannot comprehend the abstract definitions of grammar, arithmetic, and geometry, and that he recites them as he would the words of an unknown language. Only the concrete is accessible to him. When the concrete instances have been sufficiently multiplied he will unconsciously deduce from them the abstract generalities. Mathematics ought therefore to be taught experimentally, especially at first, for, contrary to current opinion, it is an experimental science.

The work of Bertrand was published when the agitation which led to the new curricula of 1902 was at its height. It sets forth his views as to French conditions and the ideal secondary school, and devotes an important and interesting chapter to the topic, "The Basis—Mathematics." A citation or two will show his views on the topic discussed in the present paper:

Mathematics ought not to be taught exclusively in the antique method as a pure science; but, according to the modern spirit, as a science at once pure and applied. We have seen with what care mathematics must be made to spring from the crude empiricism of the early years; no less systematic and minute pains must be taken to connect it with scientific experience. I pre-judge nothing as to the origin of the mathematical notions; it is the business of metaphysicians to discuss the question whether they arise from experience or are innate in the mind. But this I do know, that it is extremely regrettable to aggravate their abstract character still further by separating them from spontaneous experience, and from scientific experience.

The application of algebra to geometry, the application of mathematics to mechanics, are so evident that it is useless to dwell on them. But nothing could be more useful than to show that mathematical deduction shows points of resemblance between the most diverse phenomena—physical, chemical, biological—which direct observation could never have seized, and to show that heterogeneous facts satisfy the same numerical laws, the same geometric and mechanical relations. Mathematical law is the magic key which fits itself to the most complicated locks and, without violence or fracture, opens all doors. The mathematician is more than the auxiliary of the physicist, the chemist, the biologist, the sociologist, the moralist; they are, so to speak, his purveyors; they give him their tangible and visible realities by means of which his abstractions take shape and become themselves visible and tangible. Far from us be that narrow mathematicism which mumbles its theorems like a good dame mumbles her rosary—the one saying for each bead a little prayer, the other a meager demonstration. (P. 206.)

Similar ideas are advanced by one of the mathematical leaders of France, Mr. Jules Tannery, in the article cited above, which is noteworthy in more than one respect:

Is it credible that children thirteen or fourteen years old have a natural taste for logical abstractions, for empty ratiocinations, for demonstrations which seem much less evident to them than the things to be demonstrated? Without doubt, they must be trained to reason correctly, but to reason about realities, or at least to reason about models or images which approximate reality, which are simplifications of what they see, of what they touch. They must be made to experience how, according to Descartes, geometry facilitates all the arts. How shall I make this drawing? How measure this field?

After an illustration of how theory and practice can be made to go hand in hand (triangle and the determination of the distance of an inaccessible object), he proceeds to add:

They [the pupils] will have to reason about things; they must be trained to regard things, to eliminate this or that characteristic which is of no geometric import; to see things in their geometric aspect; to reproduce them by drawing; to gain more exact knowledge of them by measurement. Far from teaching pupils to despise intuition, this very intuition must be developed; they must be shown that they have it, and made little by little to gain confidence in themselves.

As a scientific example, our author takes up the volume of prisms, characterizing the reasoning which establishes the equivalence of oblique and right prisms as suited to "be kept in an historical museum as evidence of how intelligent our ancestors were." He suggests two means of replacing the proof. The one (mediocre) consists of cutting the two prisms into thin slices, or making the prisms out of disks of paper. With such models the theorems can be made "clear as day" to the pupils:

The second procedure, which is excellent, but demands a marked effort, consists in learning some integral calculus before studying the measurement of these volumes. Integral calculus! In the secondary school!! Yes; I am not joking. The effort needed to learn what a derivative is, an integral, and how by means of these admirable tools surfaces and volumes can be evaluated, is certainly less than the effort heretofore demanded of a child to establish the equivalence of oblique and right prisms, of two pyramids (the staircase figure, you know that is so tiresome to make), then the insupportable volumes of revolution; even today I do not know the expression for the volume generated by a segment of a circle turning about a diameter. . . .

To teach what is needed of the differential and integral calculus and of analytic geometry will require going slowly, perhaps eight or ten lessons. Do not tell me that the pupils will not understand! Why, then, do they understand what they are taught today about the volumes just mentioned? After these lessons, a quarter of an hour will suffice to establish the expressions for all the volumes of elementary geometry. And think besides of the world of ideas which will open before the pupil; of the multitude of applications which he can make.

To sum up: We have read the words of the head of the mathematical department of one of the leading higher institutions of France (Tannery); of a well-known mathe-

matician and worker in the pedagogy of mathematics (Laisant); of a professor of philosophy (Bertrand); of the head of the Pasteur Institute (Duclaux); of a general writer on a wide range of topics (Le Bon); The are all animated by the same spirit and urge: (a) the fundamental importance of the *proper* study of mathematics; (b) the concrete origin and the experimental relations of the subject; (c) the deadening effects of teaching on an abstract basis; and (d) the salutary results of beginning with the concrete.

In an address on "The General Definitions of Mathematics" delivered at the Musée Pédagogique, January 21, 1904, Mr. H. Poincaré made remarks of the same trend, advising, for example, the definition of parallels by a square sliding along a ruler and the use of the pantograph in the theory of similarity.

No recent French publication has come to my notice defending the contradictory assertions, so that it seems safe to say that the French expressions of theoretic thoughts on the teaching of mathematics trend today in the same general direction as the American movement for the introduction of what have been called "laboratory methods" into teaching of mathematics. The French views cited relate exclusively to the *method* of arranging and developing the subject-matter; with us the term "laboratory methods" connotes also something as to the mode of handling the class. In view of the strict, conventional discipline of the schools, and the fact that pupils, teachers, and textbook writers alike are subject to the absolute domination of detailed curricula, uniform throughout the republic, is it not to be expected that French writers will waste their energies in advocating anything analogous. Numerous voices have been raised against both the rigor of the discipline and the tyranny of the programs (for example, in the Inquest of 1900, and by Laisant and Le Bon), but the outlook for an early and radical change is not good; and until a change in the general policy opens the door, the consideration of details is, of course, out of the question. Even in physics and chemistry, laboratory work by pupils was unknown before the introduction of the programs of 1902.

J. W. A. YOUNG.

Education as Adjustment. By M. V. O'SHEA. New York: Longmans, Green & Co. Pp. 313.

Education as Adjustment is worth reading for three reasons. The first of these is that the author has collected from many sources a mass of valuable facts which are of sufficient concreteness and definiteness to make the volume a good book of reference. The facts are of value because they are detailed and to the point, and therefore scientific. The author follows his own precept: "The greatest need in education today is the development of a *scientific* temper among teachers and the adoption of scientific method by all who treat of educational questions (p. vii). However, as a reference book the volume might have been improved if the bibliography appended, consisting of 220 titles, could have had notes added to show what the author had found of value in each reference, in some such way as Miss Tanner has done in *The Child*. Sometimes one suspects that bibliographies are added as credentials and not for use. In the second place, in the most careful consideration that has yet appeared on the subject, it presents the case against formal discipline. The arguments are based upon biological and psychological grounds. Much use is made of facts such as Thorndike has made us familiar with (chap. 13). But it is doubtful if science has yet enough